

Mass spectra and decays of ground and orbitally excited $c\bar{b}$ states in non relativistic quark model

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The complete spectrum of $c\bar{b}$ states is obtained in a phenomenological non relativistic quark model(NRQM), which consists of a confinement potential and one gluon exchange potential (OGEP) as effective quark - antiquark potential. We make predictions for the radiative decay (E1 and M1) widths and weak decay widths of $c\bar{b}$ states in the framework of NRQM formalism.

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1. INTRODUCTION

The B_c state is the only bound system that consists of two heavy quarks of different flavours that offers a sound laboratory opportunity to observe both QCD and weak interaction. The ground state $B_c(1S)$ was first observed in the CDF and DO experiments (Tevatron) in two decay modes : $B_c \rightarrow J/\psi l \nu$ and $B_c \rightarrow J/\psi \pi$ [1–7], also observed by LHC experiments in the various decay modes of $c\bar{b}$ states such as : $B_c \rightarrow J/\psi \pi$ (LHCb, CMS, ATLAS), $B_c \rightarrow J/\psi \pi \pi \pi$ (LHCb and CMS), $B_c^+ \rightarrow \psi(2S) \pi^+$, $B_c^+ \rightarrow J/\psi D_s^+$, $B_c^+ \rightarrow J/\psi D_s^{*+}$, $B_c^+ \rightarrow J/\psi K^+$, $B_c^+ \rightarrow J/\psi 3\pi^+ 2\pi^-$ and $B_c^+ \rightarrow J/\psi K^+ K^- \pi^+ \pi^-$ (LHCb) [8–13]. Recently the ATLAS collaboration at the LHC has observed radially excited $c\bar{b}$ state (i.e, $B_c(2S)$) through the decay channel $B_c^\pm(2S) \rightarrow B_c^\pm(1S) \pi^+ \pi^-$ [14]. B_c mesons are predicted by the quark model to be members of the $J^P = 0^-$ pseudo scalar ground state multiplet [15]. The vector $B_c^*(1S)$ meson is the triplet state of $B_c(1S)$ which has not been observed by means of experiments to date.

The discovery of B_c state has made the families of well investigated charmonium and bottomonium complete and has given a new insight into the study of heavy quark dynamics. Since the charmed bottom meson $c\bar{b}$ is an intermediate state of the $c\bar{c}$ mesons and $b\bar{b}$ mesons, its analysis could give detailed information on the balance between the perturbative and non perturbative effects. The investigation of masses of $c\bar{b}$ states gives us an opportunity to obtain information on the nature of the strong interaction thereby it throws up an interesting issue and a tantalising problem. Since the flavour asymmetry of $c\bar{b}$ state unlike in symmetric quarkonium, forbids the annihilation of $c\bar{b}$ state into gluons, the ground state of $c\bar{b}$ state below the BD threshold, can only decay through weak interaction that provides an ideal platform to study weak decays and provides new methods for calculating the CKM matrix. Some decay channels of $c\bar{b}$ states show that bound state effects are significant in $c\bar{b}$ decays. The lighter c quark has a greater decay rate (65%) than the heavier b quark (25%). The pseudo scalar $c\bar{b}$ state decays only weakly. The $c\bar{b}$ is unique in that either one of its quarks can decay, leaving the other as a spectator or both the quarks may involve in its weak decay. Thus B_c meson can serve as a great laboratory for QCD sum rules, Heavy Quark Effective Theory, lattice QCD and potential models.

This work uses NRQM formalism to study both mass spectra and decay properties of $c\bar{b}$ states. The NRQM formalism is found to provide systematic treatment of the perturbative and non perturbative components of QCD at the hadronic scale [16–21]. The masses of the $c\bar{b}$ spectrum can be predicted using NRQM whose parameters are tuned to reproduce the spectra of the observed charmonium and bottomonium states [15, 22–26]. There are a good number of theoretical models that study leptonic, semileptonic and hadronic decay channels of $c\bar{b}$ state [27–36].

In the last two decades, the lattice QCD Monte Carlo calculations have emerged as a reliable non-perturbative method to study hadron spectra. For $q\bar{q}$ systems it has been shown unambiguously that the ground state potential is $V_{q\bar{q}} = -\frac{A_{q\bar{q}}}{r} + \sigma_{q\bar{q}}r + C_{q\bar{q}}$, with inter quark distance r [37–39], which is consistent with the standard NRQM potential of Coulombic + OGEP + linear confinement[40]. Also, from lattice QCD, the effect of the gluonic excitation in the three quark system has been investigated. It has been shown that for low-lying hadrons with excitation energy below 1 GeV, the effect of the gluonic excitations is negligible and hence quark degrees plays a dominant role in low-lying hadrons and hence resolves the absence of gluonic excitation modes in low-lying hadron spectra[41]. Also, the static three-quark potential has been studied in detail using SU(3) lattice QCD. The detailed analyses lattice QCD data of the 3q potential support the Y-ansatz[38].

In the recent work on lattice QCD, an investigation to estimate the masses of B, B_s and B_c has been carried out. The work takes into account of the effect of the u, d and s sea quarks. The highly improved staggered quark (HISQ) action for u/d,s and c quarks and non-relativistic QCD is used for b quarks. Using HISQ formalism, the mass difference and decay constant ratios between B, B_s and B_c mesons. The mass obtained for B_c mesons is

6280(10) GeV. which is a significant improvement on an earlier value of Bc mesons obtained from lattice QCD which is a strong test of lattice QCD to the B physics[42].

Our basic aim is to develop a consistent model which could reproduce both the spectra and the decay widths with the same set of parameters. The parameters in our work, are fixed from the mass spectroscopy and the same set of a parameters have been used to obtain the decay widths. From our analysis we infer that our model has the right prediction both for the mass spectrum and decay widths with the same set of parameters. It should be noted that obtaining the mass spectrum alone in accordance with the experimental results doesnot guarantee the validity of a model for describing hadronic interactions. Different potentials can reproduce the same spectra. Hence, in a given model, one must be able to calculate other observables like the decay constants, leptonic decay widths, the radiative decay widths, etc. Heavy quarkonium decays provide a deeper insight on the exact nature of the inter quark forces and decay mechanisms. For example, the leptonic decay widths are a probe of the quarkonium system provide important information complementary to level spacings. In the existing quark models, The OGEP has its origin in the exchange of a single gluon which belongs to an octet representation of the $SU(3)_c$. The OGEP is obtained from the QCD Lagrangian in the non-relativistic by retaining terms to the order of $1/c^2$. This procedure is similar to the derivation of the Fermi-Breit interaction in quantum electrodynamics[43]. In deriving OGEP, the gluon propagaors used are similar to the free photon propagators used in obtaining Ferm-Breit interaction in QED. Since the confinement of color means confinement of quarks as well as gluons, the confined dynamics of gluons should play a decisive role in determining the spectroscopy of the mesonic states. But, Fermi-Breit interaction which gives rise to the splitting for singlet and triplet states are treated as perturbation. But, the OGEP is attractive for singlet states and repulsive for triplet states, hence, nave perturbative treatment of OGEP is incorrect. This leads to further renormalization of strength of interaction for a better fit[44]. Also, the most prominent flaw of non-relativistic potential models is the neglect of gluon dynamics[16, 45–47]. Hence, it is required to obtain the mass spectrum by diagonalizing the Hamiltonian matrix.

The paper is organized in 4 sections. In sec. 2 we briefly review the theoretical background for non relativistic model, the description of radiative and weak decay widths. In sec. 3 we discuss the results and the conclusions are drawn in sec. 5 with a comparison to other models.

2. THEORETICAL BACKGROUND

2.1. The Hamiltonian

Essentially, in all phenomenological non relativistic QCD based quark models, the Hamiltonian for the quark system consists of the kinetic energy, the two-body confinement potential and OGEP. In most of these works, it is assumed that the principal binding forces of hadrons are the long range quark confining forces. These should be independent of quark spins and quark masses, depending just on the spatial separations of the constituent quarks.

In addition there exist short range forces depending on the quark spins and masses. The effective short range force stems from the one gluon exchange mechanism. The exchange of gluons can provide binding between quarks in a hadron. The Hamiltonian employed in our model [48–50] includes kinetic energy part, confinement potential and one gluon potential (OGEP) [45].

$$H = K + V_{CONF} + V_{OGEP} \quad (1)$$

The kinetic energy part (K) is the sum of the kinetic energies including the rest mass minus the kinetic energy of the center of mass motion (CM) of the total system, i.e.,

$$K = \left[\sum_{i=1}^2 M_i + \frac{P_i^2}{2M_i} \right] - K_{cm}, \quad (2)$$

with M_i and P_i as the mass and momentum of the i th quark, respectively. K_{CM} is the kinetic energy of the centre of mass motion.

The confinement potential must come ultimately from a non-perturbative treatment of QCD, whereas the residual interaction OGEP is based on perturbation theory. In phenomenological quark models the confinement potential is assumed to be harmonic oscillator potential ($V \sim r^2$) or logarithmic potential ($V \sim \ln(r)$) or linear potential ($V \sim r$). For our model we have chosen the linear potential which represents the non perturbative effect of QCD that confines quarks within the color singlet system [49, 50].

$$V_{CONF}(\vec{r}_{ij}) = -a_c r_{ij} \vec{\lambda}_i \cdot \vec{\lambda}_j \quad (3)$$

where a_c is the confinement strength and λ_i and λ_j are the generators of the color SU(3) group for the i th and j th quarks.

It should be noted that the two body confinement potential[51], has symmetric and antisymmetric terms.

$$V_{so}^{conf} = \sum_{i>j} -\frac{1}{4m^2 r_{ij}} \frac{dV_{ij}^{conf}}{dr_{ij}} \{ (r_{ij} \times p_{ij}) \cdot (\sigma_i + \sigma_j) + [r_{ij} \times (p_i + p_j)/2] \cdot (\sigma_i - \sigma_j) \} \quad (4)$$

If one includes both the symmetric and antisymmetric terms the two terms of V_{ij}^{conf} cancel each other and give an almost vanishing contribution to the single baryon/meson spectra. Also, it should be noted that the symmetric term of the V_{ij}^{conf} has opposite sign to the one in V_{ij}^{OGEP} and hence cancels the contribution from the symmetric term of V_{ij}^{conf} in single baryon/meson system. The antisymmetric term is Galilei non-invariant. It should be noted that the spin orbit term in V_{ij}^{conf} comes from the relativistic effects. The above two body potential is having problems with the long range attractive color Van der Waals force[52–55] when it is used for the two hadron system. Since the attraction between color singlet hadrons comes from the virtual excitation of the color octet dipole state of each hadron, this problem is evaded by restricting the model space to describe hadrons. Also, if the two body confinement mechanism is extended to two nucleon system, it essentially gives a zero contribution to the spin-orbit force of the N-N potential[56]. Since the mechanism which

we take for confinement, namely two body confinement, is purely phenomenological it is advisable to leave out the spin-orbit term due to confinement.

We consider a purely linear confinement potential in our calculation. The spin-orbit splittings calculated in our model[57] and in [58] for heavy quark system such as bottomonium and charmonium suggests a scalar confinement. W. Lucha *et al.*[58], Bhagyesh *et al.*[59, 60] and Bhagyesh and K. B. Vijaya Kumar [61] used a mixture of scalar and vector confinement potential to explain the mass spectra and decays of heavy quarkonia. In our calculation we use a one gluon exchange potential and a purely linear confinement potential. The one gluon exchange potential is of purely vector nature and the confinement potential is of purely scalar nature. The combination $V(r) = V_{OGEP} + V_{CONF}$ is a mixture of Lorentz scalar and Lorentz vector nature.

The one gluon exchange potential is given by

$$V_{OGEP} = -\frac{4}{3} \frac{\alpha_s}{r} + V_{SD}(r) \quad (5)$$

where the spin dependent potential V_{SD} is introduced as an additional term to the potential to take into the account the spin-orbit and spin-spin interactions, causing the splitting of the nL levels (n is the principal quantum number, L is the orbital momentum), so it has the form [24, 62, 63]

$$V_{SD}(r) = \left(\frac{L \cdot S_c}{2m_c^2} + \frac{L \cdot S_b}{2m_b^2} \right) \left(-\frac{dV(r)}{rdr} + \frac{8}{3} \alpha_s \frac{1}{r^3} \right) + \frac{4}{3} \alpha_s \frac{1}{m_c m_b} \frac{L \cdot S}{r^3} + \frac{4}{3} \alpha_s \frac{2}{3m_c m_b} S_c \cdot S_b 4\pi \delta(r) + \frac{4}{3} \alpha_s \frac{1}{3m_c m_b} [3(S_c \cdot n)(S_b \cdot n) - S_c \cdot S_b] \frac{1}{r^3} \quad (6)$$

The central part of the two-body potential due to OGEP is [45],

$$V_{OGEP}(\vec{r}_{ij}) = \frac{\alpha_s}{4} \vec{\lambda}_i \cdot \vec{\lambda}_j \left[\frac{1}{r_{ij}} - \frac{\pi}{M_i M_j} \left(\frac{M_i}{M_j} + \frac{M_j}{M_i} + \frac{2}{3} \vec{\sigma}_i \cdot \vec{\sigma}_j \right) \delta(\vec{r}_{ij}) \right] \quad (7)$$

where the first term represents the residual Coulomb energy and the second term is the chromo-magnetic interaction leading to the hyperfine splitting. σ_i is the Pauli spin operator and α_s is the quark-gluon coupling constant.

The non-central part of OGEP has two terms, namely the spin-orbit interaction $V_{OGEP}^{SO}(\vec{r})$ and tensor term $V_{OGEP}^{ten}(\vec{r})$. The spin-orbit interaction of OGEP is given by,

$$V_{OGEP}^{SO}(\vec{r}) = -\frac{\alpha_s}{4} \lambda_i \cdot \lambda_j \left[\frac{3}{8M_i M_j} \frac{1}{r^3} (\vec{r} \times \vec{p}) \cdot (\sigma_i + \sigma_j) \right] \quad (8)$$

where the relative angular momentum is defined as usual in terms of relative position \vec{r} and the relative momentum \vec{p} . Unlike the tensor force, the spin-orbit force does not mix states of different \vec{L} , since L^2 commutes with $\vec{L} \cdot \vec{S}$, \vec{L} is still a constant of motion, but L_z is not.

We use the following tensor term [64, 65]

$$V_{OGEP}^{ten}(\vec{r}) = -\frac{\alpha_s}{4} \lambda_i \cdot \lambda_j \left[\frac{1}{4M_i M_j} \frac{1}{r^3} \right] \hat{S}_{ij} \quad (9)$$

where,

$$\hat{S}_{ij} = [3(\vec{\sigma}_i \cdot \hat{r})(\vec{\sigma}_j \cdot \hat{r}) - \vec{\sigma}_i \cdot \vec{\sigma}_j] \quad (10)$$

The tensor potential is a scalar which is obtained by contracting two second rank tensors. Here, $\hat{r} = \hat{r}_i - \hat{r}_j$ is the unit vector in the direction of \vec{r} . In the presence of the tensor interaction, \vec{L} is no longer a good quantum number.

The only short-range force between the quarks is spin dependent which comes from OGEP. The spin-orbit couplings give rise to long range forces according to lattice QCD, since the interaction energy between two distant static quarks in lattice QCD involves a power-series expansion in inverse powers of the gluon quark coupling constant[45]. In QCD the Fermi-Breit corrections are large for mesons and baryons. For attractive potentials for the Dirac delta function $\delta(r)$, the wave equations have no physically acceptable solutions and lead to collapse both for the quark-antiquark and three quark system. Hence, there are models which have introduced a cut-off or smearing function to weaken the singularity. Though, the finite range function is calculable, it has to be parameterized[66]. But it has been shown that the short range spin interactions itself is strong enough to support a bound state of a single $q\bar{q}$ pair. The tensor and spin-orbit potentials (eqs 7 and 8) are of long range, in contrast to the zero range $\delta(r)\sigma_1 \cdot \sigma_2$ central potential (eq. 6)[67]. But the actual range of the $\delta(r)\sigma_1 \cdot \sigma_2$ is still unknown and hence is required to do a non-perturbative calculation (or a larger basis)[68]. Also there are other effects like suppression of the spin orbit potential from OGEP from the spin orbit potential arising from the confinement potential (they have opposite sign).

2.2. Radiative Decays

We consider two types of radiative transitions of the B_c meson: a) Electric dipole (E1) transitions are those transitions in which the orbital quantum number is changed ($\Delta L = 1$, $\Delta S = 0$). Examples of such transitions are $n^3S_1 \rightarrow n'^3P_J \gamma$ ($n > n'$) and $n^3P_J \rightarrow n'^3S_1 \gamma$ ($n \geq n'$). The strength of the electric dipole transitions is governed by the size of the radiator and the charges of the constituent quarks. The E1 partial decay width is given by [69],

$$\Gamma_{a \rightarrow b} = \frac{4\alpha}{9} \mu^2 \left(\frac{Q_c}{m_c} - \frac{Q_{\bar{b}}}{m_{\bar{b}}} \right)^2 \frac{E_b(k_0)}{m_a} k_0^3 |\langle R_b | r | R_a \rangle|^2 \begin{cases} (2J+1)/3, & {}^3S_1 \rightarrow {}^3P_J \\ 1/3, & {}^3P_J \rightarrow {}^3S_1 \\ 1/3, & {}^1P_1 \rightarrow {}^1S_0 \\ 1, & {}^1S_0 \rightarrow {}^1P_1 \end{cases} \quad (11)$$

where k_0 is the energy of the emitted photon,

$k_0 = m_a - m_b$ in non relativistic limit.

α is the fine structure constant. $Q_c = 2/3$ is the charge of the c quark and $Q_{\bar{b}} = 1/3$ is the charge of the \bar{b} quark in units of $|e|$, μ is reduced mass $\mu = \frac{m_b m_c}{m_b + m_c}$ and

$\frac{E_b(k_0)}{m_a} = 1$ in non relativistic limit.

$$\langle R_b | r | R_a \rangle = \int_0^\infty r^3 R_b(r) R_a(r) dr \quad (12)$$

is the radial overlap integral which has the dimension of length, with $R_{a,b}(r)$ being the normalized radial wave functions for the corresponding states.

b) Magnetic dipole (M1) transitions are those transitions in which the spin of the quarks is changed ($\Delta S = 1$, $\Delta L = 0$) and thus the initial and final states belong to the same orbital excitation but have different spins. Examples of such transitions are vector to pseudo scalar ($n \ ^3S_1 \rightarrow n' \ ^1S_0 + \gamma$, $n \geq n'$) and pseudo scalar to vector ($n \ ^1S_0 \rightarrow n' \ ^3S_1 + \gamma$, $n > n'$) meson decays.

The M1 partial decay width between S wave states is [69, 70]

$$\Gamma = \delta_{L_a L_b} 4\alpha k_0^3 \frac{E_b(k_0)}{m_a} \left(\frac{Q_c}{m_c} + (-1)^{S_a+S_b} \frac{Q_b}{m_b} \right)^2 (2S_a+1) \times (2S_b+1) (2J_b+1) \left\{ \begin{matrix} S_a & L_a & J_a \\ J_b & 1 & S_b \end{matrix} \right\}^2 \left\{ \begin{matrix} 1 & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & S_a & S_b \end{matrix} \right\}^2 \times \left[\int_0^\infty R_{n_b L_b}(r) r^2 R_{n_a L_a}(r) dr \right]^2 \quad (13)$$

which can be further simplified to

$$\Gamma_{M1}(a \rightarrow b + \gamma) = \frac{16}{3} \alpha \mu_{eff}^2 k_0^3 (2J_b+1) \left[\int_0^\infty R_{n_b L_b}(r) r^2 R_{n_a L_a}(r) dr \right]^2 \quad (14)$$

where $\int_0^\infty dr R_{n_b L_b}(r) r^2 R_{n_a L_a}(r)$ is the overlap integral for unit operator between the coordinate wave functions of the initial and the final meson states, m_c and m_b are the masses of the charm and bottom quarks and $\mu_{eff}^2 = \frac{m_b Q_c - m_c Q_b}{4m_c m_b}$. S_a , S_b , L_a , J_a and J_b are the spin quantum number, orbital angular momentum and total angular momentum quantum numbers of initial and final meson states respectively.

The M1 transitions contribute little to the total widths of the 2S levels, since it cannot decay by annihilation. Allowed M1 transitions correspond to triplet-singlet transitions between S-wave states of the same n quantum number, while hindered M1 transitions are either triplet-singlet or singlet-triplet transitions between S-wave states of different n quantum numbers. The allowed M1 transitions are essentially $1 \ ^3S_1 \rightarrow 1 \ ^1S_0$ and $2 \ ^3S_1 \rightarrow 1 \ ^1S_0$.

2.3. Weak Decays

The weak decays of bound state of a quark and an anti-quark, which carries heavy flavour c and b - enable us to probe the validity of the standard model of elementary particle interactions and determine several parameters of this model. A rough estimate of the B_c weak decay widths can be done by treating the \bar{b} -quark and c-quark decays independently so that B_c decays can be divided into three classes [71, 72]: (i) the \bar{b} -quark decay with spectator c-quark, (ii) the c-quark decay with spectator \bar{b} -quark, and (iii) the annihilation $B_c^+ \rightarrow l^+ \nu_l (c\bar{s}, u\bar{s})$, where $l = e, \mu, \tau$.

Parameter	Ref.[73]	Ref. [74]	Ref. [75]	Ref. [76]	Ref.[77]
m_c	1.8	1.48	1.48	1.48	1.55
m_b	5.174	5.18	4.88	4.88	4.88

Table 1: m_c and m_b for various theoretical models (in GeV).

3. Results and Discussion

3.1. Mass Spectra

The quark-anti quark wave functions in terms of oscillator wave functions corresponding to the relative and center of mass coordinates have been expressed here, which are of the form,

$$\Psi_{nlm}(r, \theta, \phi) = N \left(\frac{r}{b}\right)^l L_n^{l+\frac{1}{2}}\left(\frac{r^2}{b^2}\right) \exp\left(-\frac{r^2}{2b^2}\right) Y_{lm}(\theta, \phi) \quad (15)$$

where N is the normalising constant given by

$$|N|^2 = \frac{2n!}{b^3 \pi^{1/2}} \frac{2^{[2(n+l)+1]}}{(2n+2l+1)!} (n+l)! \quad (16)$$

$L_n^{l+\frac{1}{2}}$ are the associated Laguerre polynomials.

The wave function used in this calculation (eqn.15), is the standard form of the harmonic oscillator wave functions which has been extensively used in earlier works in atomic, and nuclear physics. The wave function is normalized and the normalization constant is given in eqn.16. The main advantage of using the harmonic oscillator wave function is that it allows the separation of the motion of the center of mass and has been extensively used to classify the spectra of baryons and mesons [78, 79] and extending to nucleon-nucleon interaction is straight forward [51, 80, 81]. If the basic states are the harmonic oscillator wave functions, then it is straightforward to evaluate the matrix elements of few body systems such as mesons or baryons. Since the basic states are the products of the harmonic oscillator wave functions they can be chosen in a manner that allows the product wave functions to be expanded as a finite sum of the corresponding products for any other set of Jacobi coordinates. It is advantageous to use the Gaussian form since in the annihilation of quark-anti quark into lepton pairs, the amplitude of the emission or absorption processes depend essentially on the overlap of initial and final hadrons and hence the overlap depends only on the intermediate distance region of the spatial wave functions which can extend up to 0.5 fm. This intermediate region can be described by potentials that are similar in this region and hence harmonic oscillator wave functions are expected to reproduce emission and absorption processes quite well.

The four parameters in our model are the mass of charm quark m_c , the mass of bottom quark m_b , the harmonic oscillator size parameter b and the quark-gluon coupling constant α_s . There are several papers in literature where the size parameter b is defined [67, 82]. The

value of b is fixed by minimizing the expectation value of the Hamiltonian for the vector meson. The confinement strength a_c is fixed by the stability condition for variation of mass of the vector meson against the size parameter b . To fit α_s , m_b and m_c , we start with a set of reasonable values and diagonalize the matrix for B_c meson. Then we tune these parameters to obtain an agreement with the experimental value for the mass of B_c meson. In literature we find different sets of values for m_c and m_b , which are listed in table 1.

The values of strong coupling constant α_s in literature are listed in table 2. The value of strong coupling constant ($\alpha_s=0.3$) used is compatible with the perturbative treatment. We use the following set of parameter values.

$$\begin{aligned} m_c &= 1480.0 \text{ MeV}; \quad m_b = 4750.0 \text{ MeV}; \\ b &= 0.350 \text{ fm}; \quad \alpha_s = 0.300; \quad a_c = 145 \text{ MeV fm}^{-1}; \end{aligned} \quad (17)$$

Parameter	Ref. [17]	Ref. [77]	Ref.[83]	Ref. [15]	Ref. [84]
α_s	0.21	0.265	0.357	0.361	0.391

Table 2: α_s for various theoretical models.

The mass spectrum has been obtained by diagonalizing the Hamiltonian in a large basis of 15×15 matrix which has not been carried out in other existing models which is a new ingredient in our model. The calculation clearly indicates that masses for both pseudo scalar and vector mesons converge to the experimental values when the diagonalization is carried out in a larger basis. In our earlier work also, we had come to the similar conclusion while investigating light meson spectrum[48, 85]. The diagonalization of the Hamiltonian matrix in a larger basis leads to the lowering of the masses and justifies the perturbative technique to calculate the mass spectrum. The calculation clearly indicates that when diagonalization is carried out in a larger basis convergence is achieved both for pseudo-scalar mesons and vector mesons to the respective experimental values.

For the case of a bound system of quark and anti-quark of unequal mass, charge conjugation parity is no longer a good quantum number so that the states with different total spins but with the same total angular momentum, such as the $^3P_1 - ^1P_1$ and $^3D_2 - ^1D_2$ pairs, can mix via the spin orbit interaction or some other mechanism. The B_c meson states with $J = L$ are linear combination of spin triplet $|^3L_J\rangle$ and spin singlet $|^1L_J\rangle$ states which we describe by the following mixing:

$$|nL'\rangle = |n\ ^1L_J\rangle \cos \theta_{nL} + |n\ ^3L_J\rangle \sin \theta_{nL} \quad (18)$$

$$|nL\rangle = -|n\ ^1L_J\rangle \sin \theta_{nL} + |n\ ^3L_J\rangle \cos \theta_{nL} \quad (19)$$

$$J = L = 1, 2, 3, \dots$$

Table 3: B_c meson mass spectrum (in GeV).

State $n^{2S+1}L_J$	This work	Ref.[86]	Ref. [24]	Ref. [87]	Ref. [15]	Ref.[77]	Ref.[17]	Ref.[88]	Ref.[89]
1^1S_0	6.275	6.247	6.253	6.260	6.264	6.270	6.271	$6.280 \pm 30 \pm 190$	6.286
1^3S_1	6.357	6.308	6.317	6.340	6.337	6.332	6.338	6.321 ± 20	6.341
1^3P_0	6.638	6.689	6.683	6.680	6.700	6.699	6.706	6.727 ± 30	6.701
$1P$	6.686	6.738	6.717	6.730	6.730	6.734	6.741	6.743 ± 30	6.737
$1P'$	6.734	6.757	6.729	6.740	6.736	6.749	6.750	6.765 ± 30	6.760
1^3P_2	6.737	6.773	6.743	6.760	6.747	6.762	6.768	6.783 ± 30	6.772
2^1S_0	6.862	6.853	6.867	6.850	6.856	6.835	6.855	$6.960 \pm 80 \pm$	6.882
2^3S_1	6.897	6.886	6.902	6.900	6.899	6.881	6.887	6.990 ± 80	6.914
1^3D_1	6.973		7.008	7.010	7.012	7.072	7.028		7.019
$1D$	6.974		7.001	7.020	7.012	7.077	7.041		7.028
$1D'$	7.003		7.016	7.030	7.009	7.079	7.036		7.028
1^3D_3	7.004		7.007	7.040	7.005	7.081	7.045		7.032
2^3P_0	7.084		7.088	7.100	7.108	7.091	7.122		
$2P$	7.137		7.113	7.140	7.135	7.126	7.145		
$2P'$	7.173		7.124	7.150	7.142	7.145	7.150		
2^3P_2	7.175		7.134	7.160	7.153	7.156	7.164		
3^1S_0	7.308			7.240	7.244	7.193	7.250		
3^3S_1	7.333			7.280	7.280	7.235	7.272		
2^3D_1	7.377								
$2D$	7.385								
$2D'$	7.408								
2^3D_3	7.410								
3^3P_0	7.492								
$3P$	7.546								
$3P'$	7.572								
3^3P_2	7.575								
4^1S_0	7.713				7.562				
4^3S_1	7.734				7.594				
3^3D_1	7.761								
$3D$	7.781								
$3D'$	7.783								
3^3D_3	7.796								
4^3P_0	7.970								
$4P$	7.943								
$4P'$	7.942								
4^3P_2	7.970								
5^1S_0	8.097								
5^3S_1	8.115								

Table 3: : (continued)

State $n^{2S+1}L_J$	This work	$n^{2S+1}L_J$	This work	$n^{2S+1}L_J$	This work
4^3D_1	8.132	9^1S_0	9.543	$12P'$	10.778
$4D$	8.154	9^3S_1	9.553	12^3P_2	10.787
$4D'$	8.155	8^3D_1	9.552	13^1S_0	11.015
4^3D_3	8.168	$8D$	9.573	13^3S_1	11.020
5^3P_0	8.254	$8D'$	9.572	12^3D_1	10.964
$5P$	8.314	8^3D_3	9.586	$12D$	10.972
$5P'$	8.312	9^3P_0	9.695	$12D'$	10.971
5^3P_2	8.337	$9P$	9.646	12^3D_3	10.977
6^1S_0	8.469	$9P'$	9.731	13^3P_0	11.192
6^3S_1	8.484	9^3P_2	9.749	$13P$	11.016
5^3D_1	8.494	10^1S_0	9.896	$13P'$	11.203
$5D$	8.516	10^3S_1	9.905	13^3P_2	11.208
$5D'$	8.517	9^3D_1	9.893	14^1S_0	11.458
5^3D_3	8.530	$9D$	9.912	14^3S_1	11.462
6^3P_0	8.621	$9D'$	9.912	13^3D_1	11.356
$6P$	8.675	9^3D_3	9.925	$13D$	11.361
$6P'$	8.674	10^3P_0	10.051	$13D'$	11.361
6^3P_2	8.696	$10P$	9.855	13^3D_3	11.365
7^1S_0	8.832	$10P'$	10.081	14^3P_0	11.623
7^3S_1	8.846	10^3P_2	10.097	$14P$	11.544
6^3D_1	8.851	11^1S_0	10.253	$14P'$	11.629
$6D$	8.872	11^3S_1	10.261	14^3P_2	11.632
$6D'$	8.873	10^3D_1	10.228	15^1S_0	12.005
6^3D_3	8.886	$10D$	10.243	15^3S_1	12.008
7^3P_0	8.981	$10D'$	10.242	14^3D_1	11.801
$7P$	9.027	10^3D_3	10.253	$14D$	11.804
$7P'$	9.029	11^3P_0	10.403	$14D'$	11.804
7^3P_2	9.051	$11P$	10.172	14^3D_3	11.806
8^1S_0	9.189	$11P'$	10.428	15^3P_0	12.175
8^3S_1	9.201	11^3P_2	10.441	$15P$	11.917
7^3D_1	9.203	12^1S_0	10.621	$15P'$	12.179
$7D$	9.224	12^3S_1	10.628	15^3P_2	12.180
$7D'$	9.224	11^3D_1	10.604	15^3D_1	12.350
7^3D_3	9.238	$11D$	10.617	$15D$	12.351
8^3P_0	9.339	$11D'$	10.616	$15D'$	12.351
$8P$	9.366	11^3D_3	10.625	15^3D_3	12.352
$8P'$	9.381	12^3P_0	10.762		
8^3P_2	9.401	$12P$	10.561		

where θ_{nL} is a mixing angle, and the primed state has the heavier mass. The values of the mixing angle for P states are $\theta_{1P} = 0.2^\circ$, $\theta_{2P} = 0.10^\circ$ and $\theta_{3P} = 0.05^\circ$

Similarly for $L = J = 2$ we have mixing of D states and the values of mixing angles for D states are $\theta_{1D} = 0.20^\circ$ and $\theta_{2D} = 0.05^\circ$.

The calculated masses of the $c\bar{b}$ states after diagonalization are listed in Table 3. Our calculated mass value for $B_c(1S)$ is 6275.75 MeV which agrees with the experimental value 6.275 GeV [90] and for $B_c^*(1S)$ is 6357.27 MeV. $B_c^*(1S)$ is heavier than $B_c(1S)$ by 81.52 MeV. This difference is justified by calculating the $^3S_1 - ^1S_0$ splitting of the ground state which is given by

$$M(^3S_1) - M(^1S_0) = \frac{32\pi\alpha_s |\psi(0)|^2}{9m_c m_b} \quad (20)$$

The mass of first radial excitation $B_c(2S)$ is 6862.88 MeV which is heavier than $B_c(1S)$ by 587.13 MeV. This value agrees with the experimental value of $B_c(2S)$ $6842 \pm 4 \pm 5$ [14]. The difference between the $B_c^*(2S)$ and $B_c^*(1S)$ masses turns out to be 540.14 MeV. Our prediction for masses of orbitally excited $c\bar{b}$ states are in good agreement with the other model calculations. Some of the states (i.e., 2^3P_0 , $2P_1$, $2P_1'$, 2^3P_2) are 50-100 MeV heavier in our model.

Vijande *et al.* have derived Coulomb strength $A_{3Q} \approx \frac{1}{2}A_{Q\bar{Q}}$ and confinement strength $B_{3Q} \approx B_{Q\bar{Q}}$ from the triply baryon spectra calculated in Lattice QCD [91–93]. They obtained a nice fit of the nonperturbative QCD results with the Cornell like potential

$$V^{3Q}(r) = -A \sum_{i < j} \frac{1}{|\vec{r}_i - \vec{r}_j|} + B \sum_{i < j} | + C \quad (21)$$

with Coulomb strength $A_{3Q} = 0.1875$ and confinement strength $B_{3Q} = 0.1374 \text{ GeV}^2$. The Coulomb strength and the confinement strength calculated in our model is $A_{Q\bar{Q}} = 0.4$ and $B_{Q\bar{Q}} = 0.1526 \text{ GeV}^2$ (i.e. coefficients of Coulomb and linear potentials) respectively. Using these values we come to the conclusion that $\frac{A_{3Q}}{A_{Q\bar{Q}}} \approx \frac{1}{2}$ and $B_{3Q} \approx B_{Q\bar{Q}}$. Vijande *et al.* obtained $\frac{A_{3Q}}{A_{Q\bar{Q}}} < \frac{1}{2}$ slightly different from $\frac{1}{2}$ as the one gluon exchange result and our result [92].

3.2. Radiative Decays

The calculation of radiative (EM) transitions between the meson states can be performed from first principles in lattice QCD, but these calculation techniques are still in their development stage. At present, the potential model approaches provide the detailed predictions that can be compared to experimental results. In our non relativistic model we consider the Magnetic dipole (M1) transitions and Electric dipole (E1) transitions of B_c meson.

We have listed the possible E1 decay modes in table 4 and given the predictions for E1 decay widths. Also we have compared our predictions with other theoretical models.

Most of the predictions for $E1$ transitions are in qualitative agreement. However, there are some differences in the predictions due to differences in phase space arising from different mass predictions and also from the wave function effects. For the transitions involving $P1$ and $P1'$ states which are mixtures of the spin singlet 1P_1 and spin triplet 3P_1 states, there exists huge difference between the different theoretical predictions. These may be due to the different $^3P_1 - ^1P_1$ mixing angles predicted by the different models. Wave function effects also appear in decays from radially excited states to ground state mesons such as $2^3P_0 \rightarrow 1^3S_1 \gamma$. The overlap integral for these transitions in our model vanishes and hence we get decay width for these transitions zero. This is due to the orthogonality condition for wave functions.

The possible radiative M1 transition modes are (1) $2^3S_1 \rightarrow 2^1S_0 + \gamma$, (2) $2^3S_1 \rightarrow 1^1S_0 + \gamma$, (3) $2^1S_0 \rightarrow 1^3S_1 + \gamma$ and (4) $1^3S_1 \rightarrow 1^1S_0 + \gamma$.

In the above (2) and (3) represent hindered transitions and (1) and (4) represent allowed transitions.

The resulting M1 radiative transition rates of these states are presented in table 5. In this table we give the calculated M1 decay widths for allowed transitions ($n^3S_1 \rightarrow n'^1S_0 + \gamma, n = n'$) and we compare the decay widths with other non relativistic quark models [15, 84, 89]. The hindered transitions are strongly suppressed in the non relativistic limit due to the orthogonality of the initial and final state wave functions. By adding relativistic effects to the wave function the hindered M1 transition rates can be enhanced.

3.3. Weak Decay and Life Time of B_c meson

In accordance with the classification given in section 2.3, the total decay width can be written as the sum over partial widths

$$\Gamma(B_c \rightarrow X) = \Gamma_1(\bar{b} \rightarrow X) + \Gamma_2(c \rightarrow X) + \Gamma_3(ann) \quad (22)$$

In the spectator approximation:

$$\Gamma_1(\bar{b} \rightarrow X) = \frac{9G_F^2 |V_{cb}|^2 m_b^5}{192\pi^3} \quad (23)$$

Calculated value of $\Gamma_1(\bar{b} \rightarrow X)$ is 9.628×10^{-4} eV and

$$\Gamma_2(c \rightarrow X) = \frac{5G_F^2 |V_{cs}|^2 m_c^5}{192\pi^3} \quad (24)$$

where V_{cb} and V_{cs} are the elements of the CKM matrix. The decay widths are calculated using $|V_{bc}| = 0.044$ [90] and $|V_{cs}| = 0.975$ [90].

Calculated value of $\Gamma_2(c \rightarrow X)$ is 7.712×10^{-4} eV.

The decay of vector meson into charged leptons proceeds through the virtual photon ($q\bar{q} \rightarrow l^+ l^-$). The 3S_1 and 3D_1 states have quantum numbers of a virtual photon, $J^{PC} = 1^{--}$ and can annihilate into lepton pairs through one photon. Annihilation widths such as

Transition	k_0 MeV	$\Gamma(i \rightarrow f + \gamma)$ keV	Ref. [77] keV	Ref. [15] keV	Ref. [24] keV	Ref. [89] keV
$1^3P_0 \rightarrow 1^3S_1\gamma$	281.22	30.67	75.5	79.2	65.3	74.2
$1P \rightarrow 1^3S_1\gamma$	329.71	49.438	87.1	99.5	77.8	75.8
$1P' \rightarrow 1^3S_1\gamma$	377.72	74.331	13.7	0.1	8.1	26.2
$1^3P_2 \rightarrow 1^3S_1\gamma$	380.55	112.754	122	112.6	102.9	126
$1P \rightarrow 1^1S_0\gamma$	411.23	31.974	18.4	0	11.6	32.5
$1P' \rightarrow 1^1S_0\gamma$	459.24	44.531	147	56.4	131.1	128
$2^3S_1 \rightarrow 1^3P_0\gamma$	258.92	7.98	5.53	7.8	7.7	9.6
$2^3S_1 \rightarrow 1P\gamma$	210.43	8.568	7.65	14.5	12.8	13.3
$2^3S_1 \rightarrow 1P'\gamma$	162.42	3.939	0.74	0	1.0	2.5
$2^3S_1 \rightarrow 1^3P_2\gamma$	159.59	6.228	7.59	17.7	14.8	14.5
$2^1S_0 \rightarrow 1P\gamma$	175.9	5.004	1.05	0	1.9	6.4
$2^1S_0 \rightarrow 1P'\gamma$	127.89	1.923	4.40	5.2	15.9	13.1
$2^3P_0 \rightarrow 1^3S_1\gamma$	726.95	0		21.9	16.1	
$2P \rightarrow 1^3S_1\gamma$	797.33	0		22.1	15.3	
$2P' \rightarrow 1^3S_1\gamma$	816.04	0		2.1	2.5	
$2^3P_2 \rightarrow 1^3S_1\gamma$	818.37	0		25.8	19.2	
$2P \rightarrow 1^1S_0\gamma$	861.56	0			3.1	
$2P' \rightarrow 1^1S_0\gamma$	897.56	0			20.1	
$2^3P_0 \rightarrow 2^3S_1\gamma$	186.81	14.987	34.0	41.2	25.5	
$2P \rightarrow 2^3S_1\gamma$	239.9	31.739	45.3	54.3	32.1	
$2P' \rightarrow 2^3S_1\gamma$	275.9	48.280	10.4	5.4	5.9	
$2^3P_2 \rightarrow 2^3S_1\gamma$	278.23	49.513	75.3	73.8	49.4	
$2P \rightarrow 2^1S_0\gamma$	274.43	47.512	13.8		8.1	
$2P' \rightarrow 2^1S_0\gamma$	310.43	68.770	90.5		58.0	

Table 4: **E1 transition rates of B_c meson.**

Transition	k_0 $\Gamma(keV)$	Ref. [89] $\Gamma(keV)$	Ref. [84] $\Gamma(keV)$	Ref. [77] $\Gamma(keV)$	Ref. [15] $\Gamma(keV)$	This work $\Gamma(keV)$
$1^3S_1 \rightarrow 1^1S_0\gamma$	81.52	0.190	0.060	0.073	0.135	0.0581
$2^3S_1 \rightarrow 2^1S_0\gamma$	34.53	0.043	0.010	0.030	0.029	0.00173

Table 5: **M1 transition rates for the B_c meson.**

This work	Experiment[90]	Ref. [71]	Ref. [24]	Ref. [94]	Ref. [95]
0.379	0.452 ± 0.033	0.47	0.55 ± 0.15	0.50	0.75

Table 6: **Comparison of life time of B_c meson (in ps).**

Parameter	Ref. [75]	Ref. [73]	Ref. [76]	Ref. [88]	This work
f_{B_c}	500	512	479	440 ± 20	439.735

Table 7: **Comparison of predictions for the pseudo scalar decay constant of the B_c meson.**

$c\bar{b} \rightarrow l\nu_l$ are given by the expression

$$\Gamma_3(ann) = \frac{G_F^2}{8\pi} |V_{bc}|^2 f_{B_c}^2 M_{B_c} \sum_i m_i^2 \left(1 - \frac{m_i^2}{M_{B_c}^2}\right) C_i \quad (25)$$

where m_i is the mass of the heavier fermion in the given decay channel. For lepton channels $C_i = 1$ while for quark channels $C_i = 3|V_{q\bar{q}}|^2$.

Calculated value of Γ_3 is 3.56×10^{-6} eV

Adding these results we get the total decay width

$\Gamma(\text{total}) = \Gamma_1 + \Gamma_2 + \Gamma_3 = 18.104 \times 10^{-4}$ eV corresponding to a life time of $\tau = 0.364$ ps.

The pseudo scalar decay constant f_{B_c} is defined by [15]

$$\langle 0 | \bar{b}(x) \gamma^\mu \gamma_5 c(x) | B_c(k) \rangle = i f_{B_c} V_{cb} k^\mu \quad (26)$$

where k^μ is the four-momentum of the B_c meson. In the non relativistic limit the pseudo scalar decay constant is proportional to the wave function at the origin and is given by van Royen-Weisskopf formula [96]

$$f_{B_c} = \sqrt{\frac{12}{M_{B_c}}} \psi(0) \quad (27)$$

The value of decay constant in the non relativistic potential model is listed in table 7.

4. Theoretical uncertainties in the predictions of the model

The goal of the present work is to obtain a reliable estimate of the masses of the $c\bar{b}$ states and the decay widths and understand the uncertainties in the calculation in the frame work of non-relativistic quark models. The main reason for using the non-relativistic model is, it gives a good account of the spectra and it is possible to separate the center-of-mass motion and the relativistic correction can be incorporated by v/c expansion. For $c\bar{b}$ system quark velocities are sizeable and hence needs significant corrections, but are still very small to make any significant changes in the spectra or decay widths and v/c corrections are very small and the non-relativistic predictions radiative decays are very accurate[97]. The non-relativistic quark models have many other basic features of the QCD and has met with great success in predicting the spectra and decay widths of hadrons as it allows direct calculations of the relevant matrix elements for each hadron.

The standard way of estimating the uncertainties in any model is to vary different parameters in the model.

Also it is known that using a larger harmonic oscillator basis increases the wave function at the origin since the higher order states mixing into the wave function can probe the short distance of the potential[85, 98]. Hence there is a slight theoretical uncertainty in the value of the oscillator size parameter (b). Also, there is theoretical uncertainty in the form of the wave function $\psi(0)$ at the origin. The $\psi(0)$ is relatively flat for linear and harmonic oscillator potentials, but it raises sharply for the Columbic potential. Ultimately, the form of the wave function at the origin has to be settled by lattice QCD calculations which will be the most reliable results as they are calculations from the first principles. But, for the orbitally excited states, the wave function at the origin vanishes in the non relativistic limit and hence the lowest order approximation vanishes. In calculating the masses and decay widths of quarkonium there are ambiguities in the value of α_s . These ambiguities are both

theoretical and experimental. In theory there is uncertainty in the scale μ to be considered in computing $\alpha_s(\mu^2)$. On the other hand in experimental side, there is uncertainty in the scale parameter Λ of the QCD.

Another source of theoretical uncertainty is the masses of the charm (m_c) and bottom quark (m_b). For the charm quark, masses of m_c used in literature are 1 GeV, 1.5 GeV and 1.8 GeV. But it should be noted that the spectra of the quarkonium are not very sensitive to the mass of the quark, but $|\psi(0)|^2$ is strongly dependent on m_c . But, since we are making use of the non-relativistic model both for spectrum and decay the choices for the masses of m_c are minimum. The m_c is constrained by the rate of the M1 transition $J/\psi \rightarrow \eta_c \gamma$. This branching ratio restricts $m_c \sim 2$ GeV. Our choice for m_c is 1.48 GeV which is the value quoted in PDG. For the b quark, the mass is again taken from PDG. For the b quark, for a fixed energy eigen value the dependence on $|\psi(0)|^2$ is minimal as the variation is little because of smaller uncertainty of the m_b .

5. Conclusions

The study highlights the mass spectra of $c\bar{b}$ meson in a non relativistic quark model. The ground state mass of $c\bar{b}$ state calculated in our model matches the experimental data. When the results for $c\bar{b}$ state mass spectrum are compared with the previous calculations, it is found that the predictions for the mass spectrum agree within a few MeV. The differences between the predictions in most cases do not exceed 30 MeV and the higher orbitally excited states are 50-80 MeV heavier in our model. The hyperfine splitting of the ground state vector and pseudo scalar $c\bar{b}$ states in our model is in good agreement with the prediction made by Penin et al. They predicted the hyperfine splitting of the vector and pseudo scalar $B_c(1S)$ mesons to be $M(B_c^*) - M(B_c) = 50 \pm 17(th) \text{ MeV}$. The ground state pseudo scalar B_c and vector B_c^* meson masses lie within the ranges quoted by Kwong and Rosner in their survey of techniques for estimating the masses of the $c\bar{b}$ ground state: i.e., $6.194 \text{ GeV} < M_{B_c} < 6.292 \text{ GeV}$ and $6.284 \text{ GeV} < M_{B_c^*} < 6.357 \text{ GeV}$.

Radiative decays are the dominant decay modes of the $c\bar{b}$ excited states having widths of about a fraction of MeV. In order to understand the $c\bar{b}$ spectrum and distinguishing exotic states, it is very essential that the masses and the radiative decay widths of $c\bar{b}$ states are accurately determined. The calculated M1 transition rates reasonably agree with the other theoretical model predictions as listed in table 5. It is clearly seen in this calculation that the relativistic effects play an important role in determining the M1 radiative transition rates, since the hindered transition rates are zero due to the wave function orthogonality in the NRQM formalism. The inclusion of these relativistic effects may enhance the hindered transition rates and reduce the allowed transition rates.

Most of our predictions for the E1 decay rates are in good agreement with the other theoretical calculations. The differences in the prediction for the decay rates in various theoretical models can be attributed to differences in mass predictions, wave function effects and singlet - triplet mixing angles. We have done an estimation of weak decay widths in the

spectator quark approximation and calculated the life time of $c\bar{b}$ state. We get about 53% branching ratio for b -quark decays, about 42% for c -quark decays and about 5% branching ratio in annihilation channel. The life time of B_c meson predicted in in othe theoreticla model is listed in table 6. The life time of $c\bar{b}$ state predicted in this calculation is found to be in good agreement with experimental value as well as with other theoretical predictions. The decay constant of $c\bar{b}$ state (f_{B_c}) has been calculated and compared with other model predictions and it is found that the decay constant is consistent with these predictions.

A simple non relativistic model employing OGEP and linear confinement potential used in this study is successful to predict the various properties of $c\bar{b}$ states and this can shed further light on their non leptonic transition rates.

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